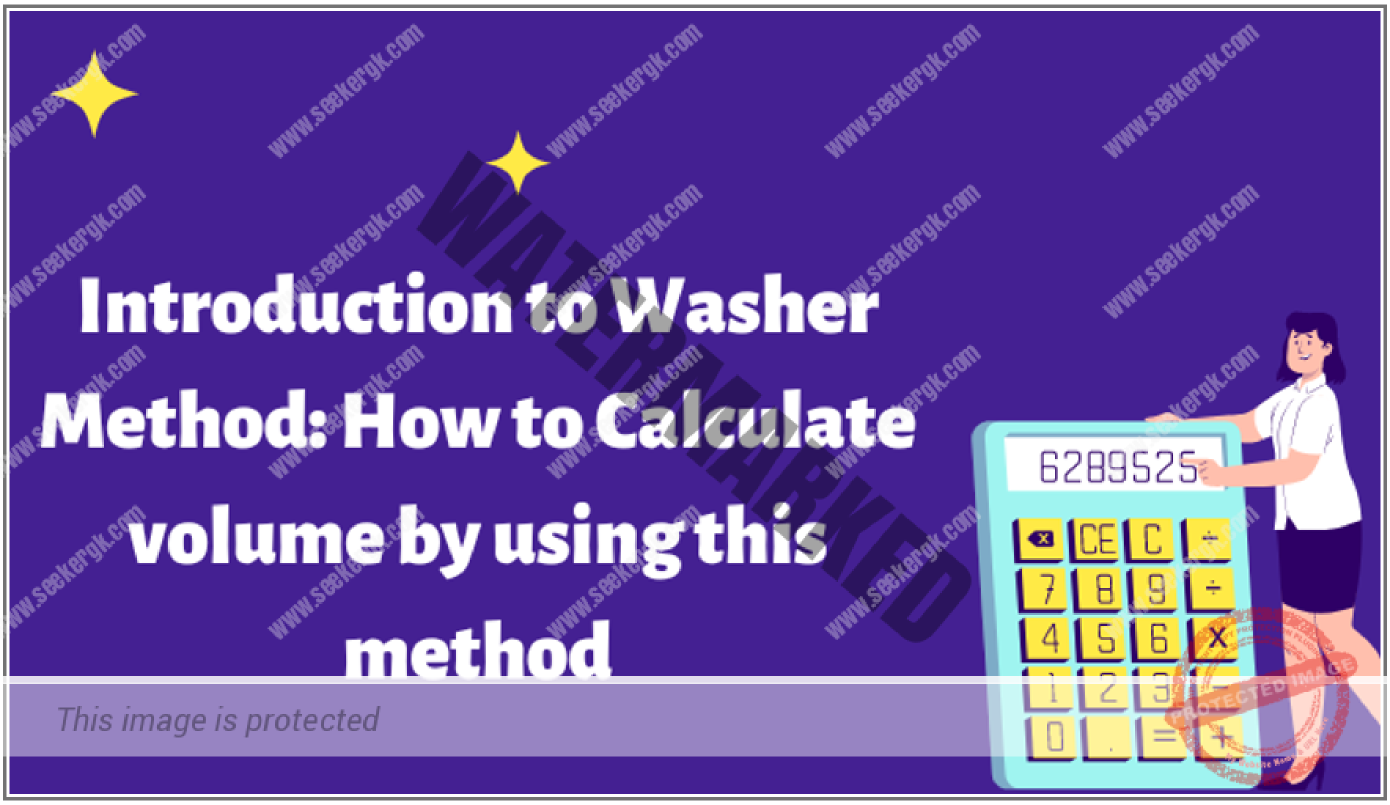
Introduction to Washer Method: How to Calculate volume by using this method



Introduction to Washer Method and How to Calculate volume by using this method

In Mathematics, there are different ways to calculate the volume of various kinds of solid shapes. With the help of integration in calculus, you can find the volume of irregular closed shapes like a round shape with a hole in the center and this shape is known as a washer or disk.

The result is represented by the rectangular section with R representing the higher function and r representing the lower function. This explains why there is a huge hole in the middle of solids generated by the space between two functions.

In this article, the basic definition of its formula and methods to calculate volume by using washer methods with detailed explanations with the help of examples will be discussed.

What is the washer method?

The washer method can be used to determine an object's volume of revolution. To accommodate items with holes, it modifies the disc approach for solid objects. The "washer technique" got its name from the way the cross sections resemble washers.

Formula:

$$\mathbf{V_y = \pi \int_c^d [F(y)]^2 - [G(y)]^2\, dy}$$

Two functions are rotated around the x-axis or the y-axis to create this shape. Create slices of the shape to determine its volume, then after determining its overall volume, take the middle space out. Calculus refers to this approach as the washing method.

How to derive the formula?

Calculating the washer's area is as easy as subtracting the area of the inner circle from the area of the outer circle.

$$A_{\text{washer}} = A_{\text{large}} - A_{\text{small}} = \pi R^2 - \pi r^2 = \pi [R^2 - r^2]$$

By integrating the area function across the range x = 0 to x = h, you can determine the total number of washers:

$$\int_0^h \pi [R^2 - r^2]\, dx$$

Now integrating with respect to x

$$\pi [R^2 - r^2] \int_0^h 1\, dx = \pi [R^2 - r^2] [x |_0^h] = \pi [R^2 - r^2] [(h) - (0)] = \pi [R^2 - r^2] [h] = \pi h [R^2 - r^2]$$

From this method, it is a derived form of a hollow cylinder: $\pi h [R^2 - r^2]$

How to Calculate Washer Method Problems?

Follow the below examples to learn how to calculate washer method problems.

Example 1

Calculate the volume of the enclosed region and you have a solid obtained by rotating the regions of parabola $x = y^2$ and the other function is $x = \sqrt{y}$ around the y-axis.

Solution:

Step 1: Draw the curves on the coordinate plane. Determine the axis of rotation.

The function $x = y^2$ and $x = \sqrt{y}$, as well as the region bounded by both curves.

The limits of integration given below for revolving both functions around the y-axis

Step 2: Equating both functions to calculate the points of intersection

$y^2 = \sqrt{y}$

Taking square on both sides it becomes

$(y^2)^2 = (\sqrt{y})^2$

$y^4 = y$

$y^4 - y = 0$

Taking common y

$y\,(y^3 - 1) = 0$

Equating both equations

$y = 0\ ,\ y^3 - 1 = 0$

$y = 0\ ,\ y^3 = 1$

$y = 0\ ,\ (y^3)^{1/3} = (1)^{1/3}$

$y = 0\ ,\ y = 1$

Therefore, roots are $c = 0$ and $d = 1$

Step 3: Now applying the formula

$F(y) = y^2$, and $G(y) = \sqrt{y}$

$V_y = \pi \int_c^d [F(y)]^2 - [G(y)]^2 \, dy$

$V_y = \pi \int_0^1 [\sqrt{y}]^2 - [(y)^2]^2 \, dy$

$V_y = \pi \int_0^1 (y - y^4) \, dy$

$V_y = \pi \int_0^1 (y) \, dy - \int_0^1 (y^4) \, dy$

$V_y = \pi [(y^2 / 2) \,\Big|_0^1 - (y^5 / 5) \,\Big|_0^1]$

$V_y = \pi [(1 / 2) - (1 / 5)]$

$V_y = \pi [(5-2 / 10)]$

$V_y = \pi [(5-2 / 10)]$

$V_y = \pi [(3 / 10)]$

$V_y = 3\pi / 10$ cubic units

To avoid such a larger calculations, you can take help from a washer method calculator to find the step-by-step solution to the given problems.

Example 2

Calculate the volume of the enclosed region and you have a solid obtained by rotating the regions of parabola $x = - y^2 + 4$ and the other function is $x = - y + 2$ around the y-axis.

Solution:

Step 1: Draw the curves on the coordinate plane. Determine the axis of rotation.

The function $x = - y^2 + 4$ and $x = - y + 2$, as well as the region bounded by both curves.

The limits of integration given below for revolving both functions around the y-axis

Step 2: Equating both functions to calculate the points of intersection

$- y^2 + 4 \quad = - y + 2$

Making it standard form

$y^2 - y - 2 = 0$

Using quadratic formula

$y = (- b \pm \sqrt{(b^2 - 4ac)}) / 2a$

$y = \{-(-1) \pm \sqrt{((-1)^2 - 4(1)(-2))}\} / 2(1)$

$y = (1 \pm \sqrt{(1 + 8)}) / 2$

$y = (1 \pm \sqrt{9}) / 2$

$y = (1 \pm 3) / 2$

$y = (1+3) / 2, y = (1 - 3) / 2$

$y = 4 / 2, y = -2 / 2$

$y = 2, y = -1$

So, the roots c = -1 and d = 2

Step 3: Now applying the formula

$F(y) = - y^2 + 4, G(y) = - y + 2$

$V_y = \pi \int^d_c [F(y)]^2 - [G(y)]^2 \, dy$

$V_y = \pi \int^2_{-1} [- y^2 + 4]^2 - [- y + 2]^2 \, dy$

$V_y = \pi \int^2_{-1} [y^4 - 8y^2 + 16 - (y^2 - 4y + 4)] \, dy$

$V_y = \pi \int^2_{-1} [y^4 - 8y^2 + 16 - y^2 + 4y - 4)] \, dy$

$V_y = \pi \int^2_{-1} [y^4 - 9y^2 + 4y + 12)] \, dy$

$V_y = \pi \int^2_{-1} (y^4)\, dy - 9 \int^2_{-1} (y^2)\, dy + 4\int^2_{-1} (y) dy + 12\int^2_{-1} dy$

$V_y = \pi\, (y^5 / 5)\, |^2_{-1} - 9\, (y^3/3)\, |^2_{-1} + 4\, (y^2 / 2)\, |^2_{-1} + 12(y)\, |^2_{-1}$

$V_y = \pi\, [(2)^5/ 5 - (-1)^5 / 5) - 9\, \{(2)^3/ 3 - (-1)^3 / 3)\} + 4\, \{(2)^2/ 2 - (-1)^2 / 2)\} + 12\{2 - (-1)\}]$

$V_y = \pi\, [29.6]$

It is a required answer.

Summary

In this article, the help of definition its formula, and derivation with detailed explanations and examples are discussed. With the help of simple integration, volume is calculated by using the washer method formula.